UCRL-ID-133885

Particle Transport and Gas Feed During Gun Injection in Spheromaks

T. K. Fowler

March 29, 1999

Lawrence Livermore National Laboratory

Work performed under the auspices of the U.S. Department of Energy by the Lawrence Livermore National Laboratory under Contract W-7405-ENG-48.

PARTICLE TRANSPORT AND GAS FEED DURING GUN INJECTION IN SPHEROMAKS

T. K. Fowler

March 29, 1999

Abstract

It is shown that ion and neutral transport during gun injection tends to equalize the density in the spheromak to that in the open-line current channel. Since a gun operating at or near the ion saturation current requires a minimum density, because of transport these gun requirements also determine a minimum density in the spheromak that increases as the field increases. Hence attaining high fields by gun injection sets lower limits on the density, which in turn limits the temperature of the plasma and increases its ohmic resistance. Estimates of these effects are given using 0-D models calibrated to CTX, as guidance to 2-D UEDGE calculations in progress. For gun power levels in SSPX and the Pulsed Spheromak reactor, we find that buildup persists to the highest field levels of interest.

1. Introduction

Ohmic heating in spheromaks tends to produce plasmas with a definite value of beta in the startup regime, dominated by Rechester-Rosenbluth heat loss due to magnetic turbulence [1]. At constant beta, if the density were also constant the temperature would rise steadily with B^2 and the integrated ohmic loss rate, $\propto \int B^2/T^{3/2}$, would saturate or decline. Hence, if the gun power were sufficient to overcome ohmic losses initially, it would continue to do so and the field energy would build up steadily until other energy loss processes take over at high temperatures.

However, experimental evidence suggests that the spheromak density is not easily controlled and the best shots producing higher fields tend to correlate with higher density, giving a less favorable temperature scaling for gun injection. Here we suggest that higher density may in fact be a requirement to produce higher fields using a gun of given power. But sufficient gun power guarantees buildup nonetheless.

Density requirements, calculated in Section 2, will be applied in Section 6 to CTX results and estimated performance in SSPX and spheromak reactors in Section 7. Section 3 reviews the magnetic field buildup equation, from Reference [2], and Sections 4 and 5 discuss ion and neutral transport, respectively.

2. Gun Density Requirements

The reason that the density must increase with the field is that a magnetized coaxial gun injecting helicity into a spheromak maintains a constant value of the gun current [2]. Assuming that the gun operates at or near the ion saturation current, sustaining this constant current requires maintaining a minimum population of ions in the open channel, by gas feed and recycling. But the channel volume, having a flux fixed by the gun, decreases as the field increases. Thus the channel density increases, and transport of ions out of the open channel into the spheromak causes the density in the spheromak to increase also. Ion and gas leakage into the private flux region, from which ions also transport into the spheromak, further increases the spheromak density.

Aside from an initial transient, the gun current I is given by [2]:

$$I = (\lambda/\mu_o)\psi \leq e\, n_E\, v_I\, (1 + g)\, \psi/B \quad , \qquad (1)$$

where ψ is the flux produced by the gun solenoid, R is the radius of the flux conserver and $\lambda = j/B$ with current density j and magnetic field B in the open-line current channel, the factor ψ/B being the channel area. The right hand side is the ion saturation current, where n_E is the ion density in the open channel, e is the electron charge, g represents secondary electrons and the ion speed v_I associated with the ion saturation current is given by:

$$v_I = 3 \times 10^5 \sqrt{\phi} \quad \text{(deuterium)} \quad , \qquad (2)$$

where ϕ is the sheath voltage in KeV (otherwise, MKS units). Solving Eq. (1) gives n_E in relation to the minimum density n_S required to sustain the current:

$$n_E \geq n_S \equiv \alpha B/R \quad , \qquad (3)$$

where here and hereafter we give densities in units of 10^{20} m $^{-3}$, and:

$$\alpha = [10^{-20} \lambda R / \mu_o e v_I (1 + g)] = 0.833 [(\lambda/\lambda_o) / (1 + g)\sqrt{\phi}], \qquad (4)$$

with Taylor eigenvalue $\lambda_o = 5/R$.

As anticipated, the minimum required density in the open channel does increase with B, assuming our estimate of the channel volume. The actual volume of the channel, calculated by Corsica, will be used in the 2 D UEDGE simulations. Since the field B in the channel is produced primarily by current stored in the spheromak, it is the increasing field of the spheromak that causes the channel volume to decrease and the density to increase.

3. Buildup of the Field

The gun circuit equation describing the buildup of the field energy E was given in Reference [2]. Neglecting losses in the flux conserver, it is given by [2]:

$$dE/dt = P - P_\Omega \qquad (5)$$

$$P = f I (V - \Delta V) \qquad (6)$$

$$\Delta V = (n_E / n_S) K T_E + 0.16 B/T_E^{3/2} \qquad (7)$$

$$P_\Omega = 2 B^2 R / T \sqrt{T_E} = 0.16 n_C R / \beta \sqrt{T_E} \quad \text{(MW)} . \qquad (8)$$

Here f is the gun efficiency (short-circuiting, etc. [2]); ΔV represents impedance in the power supply (omitted in Eq. (7)) and losses in the open-line current channel; and P_Ω is the ohmic resistive loss in the spheromak, assuming a parabolic temperature profile with edge temperature T_E (the open channel) [1]. The expression on the right is obtained from:

$$T = 12.5 \beta B^2 / n_C , \qquad (9)$$

where n_C is the density in the hot core of the spheromak. Note that the expression on the right in Eq. (8) should be replaced by that on the left at very low temperature; ohmic losses never exceed that given by $T = T_E$.

For the channel temperature T_E, we assume electron heat conduction in the open-line channel, matched to ohmic heating in the channel, which gives [2]:

$$T_E \quad = \quad 0.04\, B^{2/5} \qquad . \qquad (10)$$

Heat flow into the channel from the spheromak can yield a somewhat higher value. Using Eq. (10), both terms in ΔV are proportional to $B^{2/5}$, the first term, representing the sheath drop, being always largest, K being a dimensionless coefficient. The appearance of the midplane temperature T_E rather than ϕ in the sheath drop takes proper account of density variation at constant pressure, any increase in n due to a drop in temperature at the sheath being offset by a corresponding decrease in ϕ.

The 2D UEDGE calculations in progress can provide important information about ΔV. However, assuming good vacuum and few field errors, for gun voltages of interest (5 KeV in SSPX) we expect the open-line losses included in ΔV to be small during injection. Here we focus on the losses inside the closed confinement region, given by Eq. (8).

To calculate P_Ω, we will need expressions for the density and β in the hot core of the spheromak. We first assume Rechester-Rosenbluth heat transport, proportional to the electron thermal speed, which is very rapid, allowing us to approximate the energy balance by a quasi-steady state, giving [1]:

$$\eta j^2 \quad = \quad 3\, n T (\chi / R^2) \quad = \quad 125\, \beta^{3/2}\, (P/V_0) \quad , \qquad (11)$$

where the coefficient on the right is calibrated to CTX. Integrating over the volume V_o gives:

$$\beta \quad = \quad 0.04\, (P_\Omega / P)^{2/3} \qquad . \qquad (12)$$

Similar scalings for gyroBohm transport give beta values exceeding this up to very high fields, indicating that Rechester-Rosenbluth heat transport rather than gyroBohm dominates the buildup process. The next two sections discuss the density during buildup.

4. Charged Particle Transport During Buildup

In this section, we calculate n_C due to transport of ions out of the open channel into the spheromak. This gives the lowest density consistent with the requirements of gun injection. The actual density may be higher as discussed in later sections.

To calculate the density, we assume that particle transport out of the current channel occurs at the Rechester-Rosenbluth rate due to magnetic turbulence (inward, since the particle sources are outside). Then the fluctuations that pump magnetic energy into the spheromak at a rate proportional to the Alfven speed v_A also pump in density at a similar rate but one proportional to the ion thermal speed v_i [1]:

$$P = (v_A/\mu_o) \langle \delta B^2 \rangle_E A \qquad (13)$$

$$d/dt \int n_c = n_E v_i \langle \delta B^2 \rangle_E / B^2 \; A \quad , \qquad (14)$$

where A is the channel surface area. We solve Eq. (13) for the fluctuation level, substitute this into Eq. (14) and integrate using $v_i/v_A = \sqrt{\beta}$ and $B \approx B_o (Pt/E_o + 1)^{1/2}$ where $E = B^2 R^3$ (MJ) and subscripts (o) denote initial values. Then:

$$n_C = n_{Co} + \sqrt{\beta}\,(n_E - n_{Eo}) \approx \sqrt{\beta}\, n_E \quad , \qquad (15)$$

where the expression on the right applies if transport causes the density to grow well above initial levels.

Using Eqs. (3), (8) - (10), (12) and (15), we obtain, after a little algebra:

$$T \leq 3.53 \; (\alpha^3 P)^{-1/4} B^{6/5} R \qquad (16)$$

$$n_C \geq 0.28 \, (\alpha^5 / P)^{1/4} B^{6/5} / R \qquad (17)$$

$$P_\Omega / P \geq (4 \alpha B^{4/5} / P)^{3/4} \quad . \qquad (18)$$

From Eqs. (16) and (18), the maximum field and temperature occur at steady state ($P = P_\Omega$, by Eq. (5)) if also $n_E = n_S$ (the equalities above), giving:

$$B_{MAX} = (P/4\alpha)^{5/4} \tag{19}$$

$$T = 2.5\, B\, R/\alpha \tag{20}$$

These are the highest field and temperature that can be obtained by gun injection with ohmic heating alone, according to our model. Eq. (20) also gives the temperature obtained if injection is stopped short of steady state, after an interval in which fluctuations settle down to the levels characteristic of quiet decay ($\beta = 0.04$) [1]. For typical SSPX parameters ($P = f$ 750 MW, $R = 0.5$ m), this gives $B = 694\,(f/\alpha)^{5/4}$ and $T = 867\,(f/\alpha)^{5/4}/\alpha$ KeV, which is not at all restrictive for any reasonable value of α (order unity), even for low gun efficiency (e.g. $f = 15\%$). However, the actual field and temperature will be lower, if neutral transport feeds gas directly to the confinement regions.

5. Gas Feed

Besides ion transport, neutrals escaping from the gun channel would provide gas directly to the private flux region, and perhaps the spheromak itself. Proper treatment of gas feed and neutral transport requires the 2D UEDGE calculations. Here we only make qualitative observations, as follows.

For an optimum design with gas fed directly into the open channel, the main source of leakage is neutral propagation out of the ionization zone near the inner electrode where ion bombardment creates neutrals that recycle in the sheath. The recycle rate is quite large, given by $I_S/e \geq 10^{24}$ at the gun currents provided in SSPX. Thus, for $V_o = 0.3$ m^3 in SSPX, the density would grow at a rate $30\,(1 - g)$ / ms in our units, requiring a recycle efficiency $g > 90\%$ to avoid rapid multiplication of an initial density of 10^{20} m^{-3} in the few millisecond buildup time in SSPX. Similar constraints would apply to the Pulsed Spheromak reactor [3], with higher densities but longer times.

Ideally, recycling can be very efficient in circumstances in which the discharge is protected from direct exposure to gas other than by injection into the sheath, as in hollow cathode arcs. Then the leakage fraction, only due to the finite extent of the sheath, is given by the neutral mean free path (mm's) divided by the lateral dimension of the sheath, in this case the length along the inner electrode where the flux ψ emerges, of order 0.5 R.

Nonetheless, the conservative assumption would be that all gas is eventually ionized and contributes to density throughout the volume, giving $n_C = n_E$. Recalculating P_Ω with this assumption gives different scalings:

$$T \quad = \quad 1.66\, B^{1.92}\,(R^2/n_C^{\,3}\,P^2)^{1/5} \leq 1.66\, B^{1.32}\, R\,/\,(\alpha^3 P^2)^{1/5} \qquad (21)$$

$$n_C \quad \geq \quad \alpha B/R \qquad (22)$$

$$P_\Omega/P \quad = \quad (20\, n_C\, R\,/\,P\, B^{1/5})^{3/5} \quad \geq \quad (20\,\alpha\, B^{4/5}\,/\,P)^{3/5} \qquad (23)$$

and a new maximum field and temperature in steady state (and $n_E = n_S$ as before) :

$$B_{MAX} \quad = \quad (P\,/\,20\,\alpha)^{5/4} \quad . \qquad (24)$$

$$T \quad = \quad 0.5\, B^2\,/\,n_C \quad \leq \quad 0.5\, BR\,/\,\alpha \quad . \qquad (25)$$

Note that the maximum field in Eq. (23) is a factor $5^{5/4} = 7.5$ lower than that in Eq. (19), reflecting the assumption $n_C = n_E$ above while $n_C = 1/5\, n_E$ for the limiting case of Eq. (19), by Eqs. (12) and (15).

Eq. (24) gives the highest field that can be obtained by gun injection if ion and neutral propagation cause the density in the spheromak to track the density in the open-channel required to sustain the gun current at or near the ion saturation current. For SSPX ($P = f\,750$ MW) this gives $B_{MAX} = 93\,(f/\alpha)^{5/4}$, still not very restrictive.

Obtaining this result, or the higher field limit in Eq. (19), requires programming the gas to build up the density steadily during injection in order to avoid prohibitive ohmic losses early in the buildup. If instead the final density is present at the beginning of buildup, the density must be chosen so that $P_\Omega < P$ in Eq. (23) at $t = 0$, in which case buildup will continue until the density cannot maintain the ion saturation current in the gun, by Eq. (22). Together these requirements set yet another limit on the maximum field if the density is constant during the buildup, given by:

$$B_{MAX} \quad = \quad n_C\, R\,/\,\alpha \quad \leq \quad [P\, B_o^{\,1/5}\,/\,20\,\alpha\,] \quad , \qquad (26)$$

where B_o is the initial value produced by the startup bank in SSPX prior to sustained injection and buildup. The corresponding temperature is given by Eq. (9) with $\beta = 0.04$. For the SSPX parameters above, this gives $B_{max} \approx 37\ B_0^{1/5}\ (f/\alpha)$, which is somewhat restrictive unless the gun is more efficient than we assumed above.

6. Calibration to CTX

In Table 1 we apply the above results to the two cases from CTX studied previously [1]. In the earlier studies, we took the density as given. Here we ask whether gun requirements might have legislated these densities.

Table 1. Comparison with CTX Results

Case	R	B	n	T	α_{MAX}	ϕ_{MIN}	n_S	n/n_S
LFC	0.6	0.2	0.4	0.1	0.6	2	0.2	2
SFC	0.3	1.0	2.0	0.4	0.4	4	1.3	1.5

The approach taken is as follows. Given the sensitivity of our results to the parameter α and the uncertain volume average involved in estimating α by Eq. (4), we instead derive α from the data. Further, since the feed gas was not programmed and we do not know the recycle efficiency, we assume that the case $n_C = n_E$ applies to CTX with temperature given by Eq. (25) and we use this formula to derive α for the measured field and temperature. This formula was derived for steady state, but steady state had not been reached when the gun was crowbarred for the shots listed, as evidenced by a rise in temperature after crowbarring [1]. However, as already noted, given the field at turnoff, Eq. (25) (and Eq. (20)) also gives the maximum temperature during the decay after the gun is turned off, this being the value listed in the table. Also, again given the final field, Eq. (25) applies whether or not the density was relatively constant from the outset. The α derived in this way should be regarded as a maximum, since the formula takes the density to be that at the ion saturation current, n_S, while the actual density may be higher. Given the maximum α, we can calculate the maximum n_S, listed in the table, and also the "minimum" ϕ from Eq. (4), taking the unknown factors $(1 + g) = \lambda/\lambda_o = 1$.

The high value of ϕ in Table 1 (in KeV) is surprising, perhaps indicating that secondary electrons amplify the ion saturation current, while the ratio of the measured density to the calculated n_S, given in the last column, is supportive of the model. The fact that this ratio is greater than unity but is near unity adds credibility to our contention that gun requirements determine the relation between density and field in these experiments. We note, however, that the calculated density ratio would have been the same if we had instead assumed the case $n_C = \sqrt{\beta}\, n_E$ in deriving α, from Eq. (20), since the factor $\sqrt{\beta}$ cancels in the ratio; but the derived value of α would be 5 times larger and ϕ would be much smaller. Nonetheless, given the unlikelihood that recycling could have maintained a large ratio n_E/n_C in CTX without deliberate effort, we are inclined to accept the results of Table 1 as giving a value of:

$$\alpha \approx 0.5 \quad . \tag{27}$$

7. Summary

We have shown that requiring that the gun operate at or near the ion saturation current would set lower limits on the density in the open-line current channel, given by Eq. (3).

According to our 0 - D model, the required density increases as the field increases, so that fixing the density causes buildup of the field to cease when the field matches the ion saturation current at that density, as appears to be the case in the most successful shots in CTX (see Section 6). On the other hand, a controlled supply of gas during injection can increase the density in phase with the rising field and allow buildup to proceed to much higher field levels.

In principle the final field is effectively unlimited if gas is applied efficiently in the open-line region only, since the rate of transport of ions from the open channel into the spheromak is slower than the rate of helicity injection, causing the density in the spheromak to lag that in the open channel and yielding a very high steady state field and temperature, given in Eqs. (19) and (20). However, very good recycling efficiency is required.

The prudent plan would appear to be the middle ground, in which one does program the gas feed but concedes that the densities will equilibrate throughout, yielding the results of Eqs. (22) - (26), namely, for SSPX:

$$n_C \geq \alpha B/R \approx 1\text{x}10^{20}\, B \quad (m^{-3}) \tag{28}$$

$$T \leq 0.5\, B\, R / \alpha \approx 0.5\, B \quad (\text{KeV}) \tag{29}$$

$$B_{MAX} = (f\, P_{GUN} / 20\, \alpha)^{5/4} \approx 200\, f^{5/4} \quad (\text{tesla}) \tag{30}$$

where for SSPX we take the gun power to be 750 MW, $\alpha = 0.5$ by Eq. (27), and R = 0.5 m.

With these assumptions, the greatest leverage is through the gun efficiency factor f, which can be improved by adjusting gun coils to minimize short-circuiting in the gun [2], and control of the gas feed. To realize the steady state bound on the field, Eq. (30), requires programming the density to track the field, by Eq. (28). Then the real limit on B in SSPX is the bank energy, giving, for R = 0.5 m in SSPX:

$$B_{MAX} = 2.8\, [f\, E_{BANK}\, (\text{MJ})\,]^{1/2} \quad . \tag{31}$$

Otherwise, at fixed density, Eq. (28) becomes the bound on the field, in turn limited by the power and initial field, by Eq. (26). These results are summarized in Figure 1.

Similar considerations apply to the Pulsed Spheromak reactor [3], where again it would be desirable to simplify gas control. Using Eqs. (24) and (25), we find that obtaining the field of 27 T required for ohmic ignition would require a net injected power around 150 MW, within the range of values considered in Reference [3].

The main uncertainties, concerning the parameter α derived from CTX data in Section 6, could be resolved by UEDGE calculations of the density required to sustain the gun current for a succession of equilibria representing the buildup process.

Our results concern ohmic losses on the closed lines only, the open-line losses probably being small relative to the dynamo power during buildup, as noted in Section 3. This comment does not always apply to measurements of the decaying field, in which case dynamo voltages of a few 100 volts on open field lines -- small compared to the gun voltage -- nonetheless can cause losses in excess of those on closed lines during decay, as in the CTX experiments with a mesh flux conserver [4]. The approximate condition for closed-line losses to dominate during the decay phase is found by taking the ratio of losses on the closed and open lines:

$$I_{CLOSED} / I_{OPEN} = (\eta\lambda)_{OPEN}/(\eta\lambda)_{CLOSED}) \approx (\lambda_{OPEN}\, T / \lambda_o\, T_E) > 1 \quad . \qquad (32)$$

In the third expression we evaluated the resistivity on open lines for electron-ion collisions, though in some experimental situations electron-neutral collisions can dominate [4]. The CTX with a solid flux conserver marginally satisfied this condition for the range $T < 100$ eV which exhibited magnetic decay times $\propto T$ indicative of dominance of closed line losses. Note that the current on the open lines should be taken as $\approx I_{GUN}$, this being the threshold for the instabilities needed to drive the dynamo on the open lines; otherwise there would be no open line losses (in which case $\lambda_{OPEN} << \lambda_o$). The constancy of I_{OPEN} also gives a rate of decay of the magnetic field that is characteristic of the open-line resistivity when open-line losses dominate the decay -- linear (dI/dt constant) for electron-neutral resistivity [4] or a time constant $\propto B^{3/5}$ for electron-ion resistivity [5].

References

[1] T. K. Fowler and D. D. Hua, J. Fusion Energy **14**, 181 (1995).

[2] T. K. Fowler, "Gun Requirements to Achieve High Field Spheromaks," UCRL-ID-133245, March 4, 1999.

[3] T. K. Fowler, D. D. Hua, E. B. Hooper, R. W. Moir, L. D, Pearlstein, "Pulsed Spheromak Fusion Reactors," UCRL-JC-132109 Preprint, Accepted for publication in Comments on Plasma Physics and Controlled Fusion

[4] T. R. Jarboe, Plasma Phys. Control. Fusion **36**, 945 (1994).

[5] E. C. Morse and C. W. Hartman, DPP/APP New Orleans, Nov. 1998.

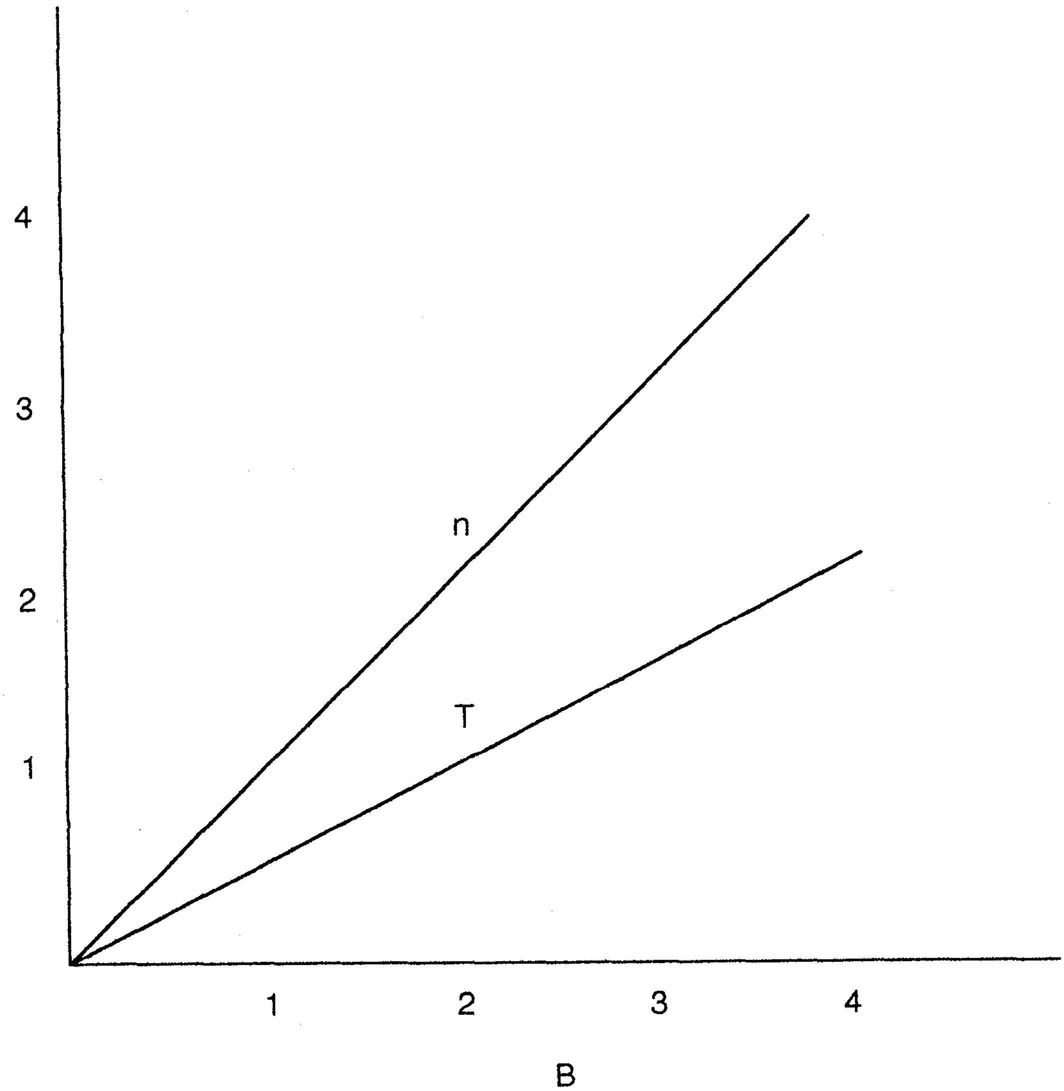

Figure 1. Predicted performance in SSPX (n in units 10^{20} m^{-3}, T in KeV, B in tesla). Maximum field is either power limited, Eqs. (26) or (30), or density limited Eq. (28), or limited by the available capacitor bank energy, Eq. (31).